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THE THEORY OF ELECTRON-RAY HIGH-FREQUENCY OSCILLATORS

G. Ya. Myakishev
26 November 1947

This work studies the propagation of modulations along an electron beam, when the modulated quantities are current, kinetic energy, and charge density, and when the propagation is caused by disturbances superimposed on a definite point of the ray.

Investigation by Kinetics of the Propagation of Modulations, Not Taking Into Consideration Interaction Between the Electrons

The kinetic equation of Vlasov (Scientific Memoirs, Moscow State University, No 75, 1945) will serve as the initial equation of our problem; it expresses in the form of a differential equation the function $f(x,\xi,t)$ describing the distribution of electrons in one dimension:

$$\frac{\partial f}{\partial t} + \xi\frac{\partial f}{\partial x} + \frac{e}{m}E(x,t)\frac{\partial f}{\partial \xi} = 0,$$

where $E(x,t)$ is a given field strength between two grids causing the disturbance of the electron beam. If the distance between the grids is small, and the time required by the electrons to travel between them is much less than the period of the potential superimposed on the grids, then it is possible to effect a transition to a dual electrical level thus:

$$E(x,t) = V(t)\delta(x),$$

where $\delta(x)$ is the delta function of Dirac. We seek the solution in the form

$$f = f_0 + f_1,$$

where f_0 is the function describing the distribution of electrons in a non-disturbed beam; this function f_0 we set to equal to:

$$f_0 = N_0\,\delta(\xi - \xi_0).$$

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Here N_0 — is the concentration of electrons in the unperturbed beam. This function is correct for large velocities of electrons in the beam, when it is possible to neglect the velocity distribution of the electrons. The function f_1 — is the small increment (that is, perturbation) caused by the disturbance, which we shall take as quite small. Therefore it is possible to neglect the effect of f_1 on the disturbing force E, in comparison with other terms.

Hence

$$\frac{\partial f_1}{\partial t}+\xi\frac{\partial f_1}{\partial x}+\frac{e}{m}V(t)\,\delta(x)\frac{\partial f_0}{\partial \xi}=0.$$

This equation can change to its equivalent,

$$f_1(x,\xi,t)=\sigma_0 f_1(x,\xi,t_0)-\frac{e}{m}\frac{\partial f_0}{\partial \xi}\int_{t_0}^{t}V(\tau)\,\sigma\,\delta(x)\,d\tau,$$

where σ_0 and σ — are displacement operators of x.

Operator σ_0 is defined by the operation $x\to x-\xi(t-t_0)$ [illegible] with σ is similarly defined thus: $\sigma - x\to x-\xi(t-\tau)$.

Let us introduce a disturbance (perturbation) at the moment $t_0=0$ and at a point $x=0$. Then $\sigma_0 f_1(x,\xi,0)=0$, since at this moment the increment is absent after calculation of the disturbance. After integrating, we obtain,

$$f_1(x,\xi\,t)=-\frac{e}{m}\frac{\partial f_0}{\partial \xi}\cdot\frac{1}{\xi}\cdot V\left(t-\frac{x}{\xi}\right).$$

This expression is distinct from zero for $x>0$ and $t>\frac{x}{\xi}$. The density of any quantity $\psi(x,\xi)$, in connection with an electron beam, is as follows:

$$\rho_\psi=\int_{-\infty}^{+\infty}\psi(x,\xi)\,f(x,\xi,t)\cdot d\xi.$$

Setting the psi function equal to the following (respectively: charge, current, energy) in succession:

$$\psi=e,\ e\xi,\ \frac{m\xi^2}{2},$$

we obtain correspondingly the density of the charge, $\rho(x,t)$, the density of the current, $j(x,t)$, and the density of the kinetic energy, $W(x,t)$, thus:

$$\rho(x,t)=N_0 e\left\{1+\frac{e}{m}\frac{x}{\xi_0^3}\frac{\partial V(\theta)}{\partial t}-\frac{e}{m}\frac{1}{\xi_0^2}V(\theta)\right\}$$

$$j(x,t)=N_0 e\,\xi\left\{1+\frac{e}{m}\frac{x}{\xi_0^3}\frac{\partial V(\theta)}{\partial t}\right\};$$

$$W(x,t)=\frac{N_0 m\xi_0^2}{2}\left\{1+\frac{e}{m}\frac{1}{\xi_0^2}V(\theta)+\frac{e}{m}\frac{x}{\xi_0^3}\frac{\partial V(\theta)}{\partial t}\right\}$$

(where $\theta=t-\frac{x}{\xi_0}$).

Integration was effected in accordance with the principle of integrating the derivative delta function. The expression for the density of the current coincides with the expression obtained by Webster (J. Appl. Phys. 10, 501. 1939) from different calculations for $V(t)=V_0\sin\omega t$.

<u>Investigation by Kinetics of Concentrated Beams, Taking Into Consideration Interaction Between the Electrons</u>

As an initial system of equations, in the study of Coulomb's force, we use the equation for the distribution function, in agreement with Poisson's equation:

$$\left.\begin{aligned}&\frac{\partial f}{\partial t}+\xi\frac{\partial f}{\partial x}+\frac{e}{m}E(x,t)\frac{\partial f}{\partial \xi}+\frac{e}{m}V(t)\;\delta(x)\frac{\partial f}{\partial \xi}=0\\&\frac{\partial E(x,t)}{\partial x}=4\pi e\int_{-\infty}^{+\infty}f(x,\xi,t)\;d\xi+4\pi\rho_0\end{aligned}\right\}\quad(1)$$

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where [illegible] is the field caused by the electrons and also by possible external charge ρ_0.

We shall solve the system (after setting $f=f_0+f_1$ and $E=E_0+E_1$) where the null-approximation includes the influence on the beams of the proper field of a nondisturbed beam and also of external fields, excluding the field arising from considerations of the action of the grids. The problem is complicated by the scattering of the beam caused by the presence of the space charge, but this does not appear to be the principal part of the problem and may be compensated by the focusing arrangement. What interests us is the field caused by the excess or by the deficiency of the space charge relative to the average value, i.e., the value during absence of the disturbance of the two grids. It is possible to calculate what field of this average value is compensated by a charge of positive ions.

Then $f_0=N_0\ \delta(\xi-\xi_0)$ as before, and $E=0$; and equations (1), after disregarding terms of the second order of smallness, are equivalent to:

$$\frac{\partial f_1}{\partial t}+\xi\frac{\partial f_1}{\partial x}+\frac{e}{m}E_1(x,t)\frac{\partial f_0}{\partial \xi}+\frac{e}{m}V(t)\cdot\delta(x)\cdot\frac{\partial f_0}{\partial \xi}=0$$

$$\frac{\partial E_1}{\partial x}=4\pi e\int_{-\infty}^{+\infty}f_1(x,\xi,t)\,d\xi=4\pi\ \rho_1(x,t).$$

We shall write down the first equation in the equivalent form:

$$f_1=\sigma_0\ f_1(x,\xi,t_0)-\frac{e}{m}\frac{\partial f_0}{\partial \xi}\int_{t_0}^{t}V(\tau)\ \sigma\,\delta(x)\,d\tau-\frac{e}{m}\frac{\partial f_0}{\partial \xi}\int_{t_0}^{t}\sigma\,E_1(x,\tau)\,d\tau,$$

where σ_0 and σ are operators. Again let us set $t_0=0$, and, by analogy, let us set the first term of the first part equal to zero.

Further:

$$f_1(x,\xi,t)=-\frac{e}{m}\frac{\partial f_0}{\partial \xi}\frac{1}{\xi}V\left(t-\frac{x}{\xi}\right)-\frac{e}{m}\frac{\partial f_0}{\partial \xi}\int_0^t E_1[x-\xi(t-\tau),\tau]\,d\tau.$$

Multiplying the right and left parts of the equation by e and integrating with respect to ξ from $-\infty$ to $+\infty$:

$$\rho_1(x,t)=\frac{N_0e^2}{m}\left\{\frac{x}{\xi_0^3}\frac{\partial V(\theta)}{\partial t}-\frac{1}{\xi_0^2}V(\theta)\right\}+\frac{N_0e^2}{m}\int_0^t(\tau-t)\frac{\partial E_1(u,\tau)}{\partial u}\,d\tau,$$

where $\qquad u=x-\xi_0(t-\tau);$

or

$$\rho_1(x,t)=\frac{N_0e^2}{m}\left\{\frac{x}{\xi_0^3}\frac{\partial V(\theta)}{\partial t}-\frac{1}{\xi_0^2}V(\theta)\right\}-\omega_0^2\int_0^t(t-\tau)\,\rho_1(u,\tau)\,d\tau,\qquad(2)$$

where $\omega_0^2=\frac{4\pi N_0e^2}{m}$ is the square of the natural frequency of the electron circuit.

Therefore, the integral equation of Walter is obtained for $\rho_1(x,t)$ with a simple integrand. Its peculiarity is the dependence of ρ_1 not upon x, but upon $x-\xi_0(t-\tau)$. Let us designate $-\omega_0^2=$ [illegible] and the first term of the right part of equation (2) by $\phi(x,t)$. We shall find the solution, after assuming that $\phi(x,t)$ depends upon x only in the following combination: $\phi\left(t-\frac{x}{\xi_0}\right)$. This solution of course will not be accurate since the first term of ϕ has x. In this case, by changing t in ϕ to τ and x to u, ϕ becomes a function of $t-\frac{x}{\xi_0}$.

We seek the solution in the form of a series:

$$\rho_{11}(x,t)=\phi(\theta)+\sum_{n=1}^{\infty}\lambda^n\psi_1(x,t).$$

After placing the series in the initial equation and collecting terms of the same power in λ, we find $\psi_n(x,t)$.

$$\Psi_1(x,t)=\int_{t_0}^{t}(t-\tau)\,\phi\left(\tau-\frac{x}{\xi_0}\right)d\tau=\phi\left(t-\frac{x}{\xi_0}\right)\int_0^t(t-\tau)\,d\tau;$$

$$\Psi_2(x,t)=\int_0^t(t-\tau)\,\psi_1(x,\tau)\,d\tau=\phi\left(t-\frac{x}{\xi_0}\right)\int_0^t K_2\,d\tau;$$

$$\Psi_3(x,t)=\phi\left(t-\frac{x}{\xi_0}\right)\int^t K_3\,d\tau$$

Here the characteristic part is no longer dependent ... $\rho_{11}(x,t)$. $K, K_3, \dots$, etc., are integrations of the integrand.

The resolvent of this integrand will be $\dfrac{\sin\omega_0(t-\tau)}{\omega_0}$.

$$\rho_{11}(x,t)=\frac{N_0e^2}{m}\left\{\frac{x}{\xi_0^3}\frac{\partial V(\theta)}{\partial t}-\frac{1}{\xi_0^2}V(\theta)\right\}-\omega_0^2N_0\frac{e^2}{m}\left\{\frac{x}{\xi_0^3}\frac{\partial V(\theta)}{\partial t}-\frac{1}{\xi_0^2}V(\theta)\right\}\int^t\frac{\sin\omega_0(t-\tau)}{\omega_0}\,d\tau=N_0\frac{e^2}{m}\left\{\frac{x}{\xi_0^3}\frac{\partial V\theta}{\partial t}-\frac{1}{\xi_0^2}V(\theta)\right\}\cos\omega_0t$$

Let us place ρ_{11} in equation (2).

We obtain then the function $F(x,t)$.

$$F(x,t)=\rho_{11}+\omega_0^2\int^t(t-\tau)\,\rho_{11}(x,\tau)\,d\tau-\phi(x,t)=-\frac{2N_0e^2}{m\xi_0^2}\frac{\partial V(\theta)}{\partial t}\left\{t+\frac{\sin\omega_0t}{\omega_0}\right\}.$$

The unknown $\rho_1=\rho_{11}(x,t)+\rho_{12}(x,t)$. The integral equation for ρ_{12} possesses the form:

$$\rho_{12}(x,t)+\omega_0^2\int_0^t(t-\tau)\,\rho_{12}(x,\tau)\,d\tau=F(x,t).$$

Now in $F(x,t)$, x enters only in the combination $t-\dfrac{x}{\xi_0}$.

After solving, by analogy, this equation, we find:

$$\rho_{12}=\frac{N_0e^2}{m\xi_0^2\omega_0}\cdot\frac{\partial V(\theta)}{\partial t}\sin\omega_0t-\frac{N_0e^2}{m}\frac{t}{\xi_0^2}\cdot\frac{\partial V(\theta)}{\partial t}\cos\omega_0t.$$

Consequently,

$$\rho(x,t)=N_0e\left\{1+\frac{e}{m}\frac{x}{\xi_0^3}\frac{\partial V(\theta)}{\partial t}\cos\omega_0t+\frac{e}{m}\frac{1}{\xi_0^2}\frac{1}{\omega_0}\frac{\partial V(\theta)}{\partial t}\sin\omega_0t-\frac{e}{m}\frac{t}{\xi_0^2}\frac{\partial V(\theta)}{\partial t}\cos\omega_0t-\frac{e}{m}\frac{1}{\xi_0^2}V(\theta)\cos\omega_0t\right\}.$$

If we set $\omega_0t\ll1$, that is, if the concentration is small or the disturbance is not propagated far from the grids, then we obtain the previous solution for the case where the electrons do not interact upon each other.

Employing the ratios $\dfrac{\partial\rho}{\partial t}=-\dfrac{\partial j}{\partial x}$ and assuming $V(t)$ to be expanded into a Fourier series, we find the density of the current j:

$$j(x,t)=N_0e\xi_0\left\{1+\frac{e}{m}\frac{x}{\xi_0^3}\frac{\partial V(\theta)}{\partial t}\cos\omega_0t+\frac{e}{m}\frac{t}{\xi_0^2\omega_0}V(\theta)\sin\omega_0t+\frac{e}{m}\frac{1}{\omega_0\xi_0^2}\frac{\partial V(\theta)}{\partial t}\sin\omega_0t-\frac{e}{m}\frac{x}{\xi_0^3}\omega_0V(\theta)\sin\omega_0t-\frac{e}{m}\frac{t}{\xi_0^2}\frac{\partial V(\theta)}{\partial t}\cos\omega_0t\right\}.\quad(4)$$

This is the solution for an electron beam propagated from two grids and completely unlimited. For a finite beam of length l, this solution is true for $t<\dfrac{l}{\xi_0}$.

<u>The Case of a Finite Beam of Length l</u>

Webster solved an analogous problem by a different method. He introduced

for j, when $V(t) = V_0 \sin\omega_0 t$, the expression:

$$j = j_0 \cdot \left\{ 1 + \frac{e}{m}\frac{\omega}{\omega_0}\frac{1}{\xi_0} \cos\omega\theta \, \sin\frac{\omega_0}{\xi_0} x \right\},$$

[illegible] whereupon he assumed that amplitudes of the electron loops, resulting from the disturbance, are everywhere identical. He took into account the influence of the field on an individual electron from just one loop (antinode), disregarding the influence of the remaining loops. If we make a similar assumption, then the solution obtained must be changed.

Let us determine j at a point x at the moment of arrival there of the disturbance, that is, for $t = \frac{x}{\xi_0}$, since the average speed of the electron beam is ξ_0. With further increase in time t, we shall assume no variation in current j at the point x because the electron field has moved ahead. Then replacing t by $\frac{x}{\xi_0}$ in equation (4) everywhere, except the terms $V(\theta)$ and $\frac{\partial V(\theta)}{\partial t}$ (since the variation in j at x with the flow of time takes place because of grid action), we obtained the expression of Webster. Consequently, for each x it is correct at the moment of propagation of the disturbance to the point x. For large values of time, this expression is very approximate.

It is possible, although not strictly, to obtain for ρ and j a more accurate expression from the formulas obtained for an infinite beam. In addition, we shall study ρ and j at the end of the beam, practically the most important point. Let us find the expression for $\rho(l,t)$. To obtain the solution, analogous to Webster's solution, it is necessary [illegible] t by $\frac{l}{\xi_0}$ in equation (3). But, in addition, we do not take into account that the field of the beam has acted previously on the electrons forming ρ at point l, with the beam possessing a variable length from l to 0. If we set in (3) $t = \frac{2l}{\xi_0}$, this will mean that the field of a beam of length l has acted previously upon the electrons forming ρ at point $z = l$. Actually, the length varies from l to 0 and consequently one solution will be larger, but another less, than the true solution. We shall obtain a more accurate solution by taking the average of these two. Therefore: $\rho(l,t) =$

$$N_0 e \left\{ 1 + \frac{1}{2}\frac{e}{m}\frac{1}{\xi_0^2}\frac{1}{\omega_0}\frac{\partial V(\theta')}{\partial t} \sin\omega_0\frac{l}{\xi_0} - \frac{1}{2}\frac{e}{m}\frac{1}{\xi_0^2} V(\theta) \cos\omega_0\frac{l}{\xi_0} - \frac{1}{2}\frac{e}{m} \frac{l}{\xi_0^3}\frac{\partial V(\theta')}{\partial t} \cos\omega_0\frac{2l}{\xi_0} + \frac{1}{2}\frac{e}{m}\frac{1}{\omega_0}\frac{1}{\xi_0^2}\frac{\partial V(\theta')}{\partial t} \sin\omega_0\frac{2l}{\xi_0} - \frac{1}{2}\frac{e}{m}\frac{1}{\xi_0^2} V(\theta') \cos\omega_0\frac{2l}{\xi_0} \right\}.$$

(here we have placed $\theta' = t - \frac{l}{\xi_0}$).

For small l and ω_0, this solution transforms into the solution for the case where there is no interaction between the electrons. Analogously, it is possible to write the expression for $j(l,t)$:

$$j(l,t) = N_0 e \xi_0 \left\{ 1 + \frac{1}{2}\frac{e}{m}\frac{1}{\omega_0 \xi_0^2}\frac{\partial V(\theta')}{\partial t} \sin\omega_0\frac{l}{\xi_0} + \frac{1}{2}\frac{e}{m}\frac{1}{\xi_0^2 \omega_0}\frac{\partial V(\theta')}{\partial t} \sin\omega_0\frac{2l}{\xi_0} + \frac{1}{2}\frac{e}{m}\frac{l}{\xi_0^3}\omega_0 V(\theta') \sin\omega_0\frac{2l}{\xi_0} - \frac{1}{2}\frac{e}{m}\frac{l}{\xi_0^3}\frac{\partial V(\theta')}{\partial t} \cos\omega_0\frac{2l}{\xi_0} \right\}.$$

In conclusion I express my deep gratitude to Professor A. A. Vlasov who proposed this theme and helped by his advice.

- END -